Decision Theory:
Ambiguity and Variational Preferences

BAMS project / course insight - Decision Theory

Abstract.

Expected utility evaluates an uncertain prospect using a single probability distribution. When that distribution is poorly known or the underlying model is distrusted, a decision maker may consider several plausible beliefs. This project asks how such ambiguity changes choice, how it can be represented mathematically, and what the resulting representations say about confidence in an economic model.

A starting point could be a finite set of states and the maxmin expected utility model. From there, the project explores variational preferences, which evaluate an act by minimizing expected utility plus a penalty over possible probability distributions:

$$V(f) = \inf_{P \in \mathcal{P}} \{\mathbb{E}_P[u(f)] + \alpha(P)\}.$$

Here u describes utility over outcomes and α assigns a penalty to each prior. The inquiry connects preferences over uncertain acts with convex analysis, statistical divergences and robust economic decisions.

Prerequisites / Difficulty.

- Expected utility, preferences and basic probability; prior Decision Theory is recommended.
- Linear algebra, continuity, convexity and constrained optimization.
- Willingness to develop the convex analysis needed for representation results.

Intermediate to advanced, with an accessible finite-state starting point. The depth depends on the questions pursued. A focused investigation can compare models and derive symbolic examples; a more advanced version can study axiomatic representation and its proof. Convex duality and more general state spaces can be explored progressively.

Starting materials.

Itzhak Gilboa (2009), *Theory of Decision under Uncertainty*.

Itzhak Gilboa and David Schmeidler (1989), "Maxmin expected utility with non-unique prior", *Journal of Mathematical Economics*.

Fabio Maccheroni, Massimo Marinacci and Aldo Rustichini (2006), "Ambiguity Aversion, Robustness, and the Variational Representation of Preferences", *Econometrica*.

Peter Klibanoff, Massimo Marinacci and Sujoy Mukerji (2005), "A Smooth Model of Decision Making under Ambiguity", *Econometrica*.

Possible directions.

- **From risk to ambiguity.** Clarify the distinction between uncertainty described by a given probability distribution and uncertainty about which distribution to use. Examine the Ellsberg paradox and the preference patterns it highlights. Distinguish risk aversion from ambiguity aversion, and identify which features of standard expected utility are challenged.

- **Comparing ambiguity models.** Study maxmin expected utility and the economic interpretation of a set of priors. Use finite-state acts to examine how enlarging that set changes evaluations. A further comparison with smooth ambiguity preferences could investigate how beliefs about models and attitudes toward ambiguity enter choice in different ways.

- **Understanding the variational representation.** Investigate the role of the penalty α, including its convexity, normalization and regularity. Explain why the penalty enters with a plus sign inside the minimization: a larger penalty makes a prior less influential in the pessimistic evaluation. Recover maxmin preferences by assigning zero penalty to an admissible set of priors and infinite penalty outside it.

- **Divergences and distrust of a reference model.** Introduce penalties based on discrepancies between probability distributions. Compare relative entropy with another choice, such as total variation or a squared-distance penalty, initially on a finite state space. Derive symbolic examples showing how the penalty changes the minimizing prior and the evaluation of an act. Discuss what each specification assumes about model uncertainty.

- **A representation theorem as a possible target.** Select a precise result linking properties of preferences to a variational representation. State its assumptions, explain their behavioral meaning and develop the proof strategy and key steps. A finite-state formulation can provide a starting point for understanding how concavity and supporting hyperplanes lead to a penalty representation.

- **Further exploration.** With more time, investigate links with convex risk measures, robust control or statistical model selection. Other directions include comparing additional divergences, studying the interpretation of penalty parameters, or asking what changes when decisions are made over time and beliefs must be updated.

Expected outcome.

The project offers a way to connect decision theory with convex optimization and the mathematics of probability distributions. It can develop an understanding of what an ambiguity model assumes about beliefs, how a penalty represents attitudes toward model uncertainty, and why different specifications can lead to different economic decisions.

A focused theoretical report could compare expected utility, maxmin and variational preferences through finite-state examples and selected proofs. The inquiry can expand with the time devoted to it. A more extensive version could reconstruct a representation argument, compare two penalty families in depth, or develop an application to robust economic decision making.