The Wire Paradox:
From Electromagnetism to Relativity

BAMS project - Physics II / Electromagnetism

Abstract.

A neutral wire deflects a moving charge through a magnetic force. In the charge's instantaneous rest frame, that magnetic force vanishes. Where does the interaction come from? You will follow this puzzle from familiar electromagnetism to a relativistic description of space, time and fields.

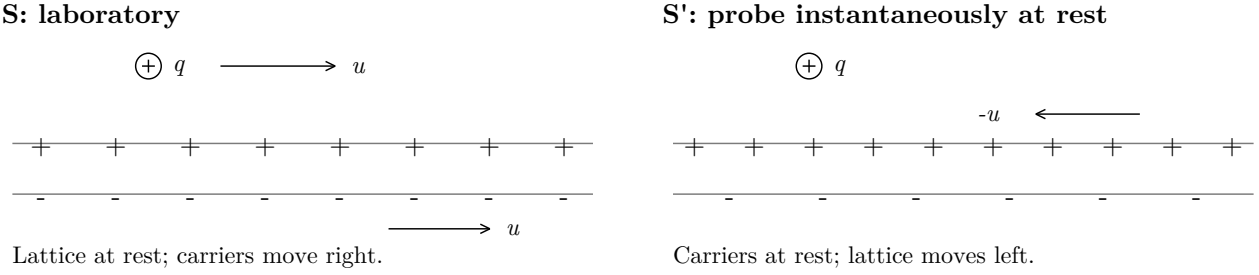
Prerequisites / Difficulty.

- Physics II: electric and magnetic fields, Gauss's and Ampère's laws, the Lorentz force and Maxwell's equations.
- Vectors, basic algebra and calculus. Programming is optional.

Intermediate. You will learn special relativity during the project, using motion along one axis. Tensor methods are optional.

What we will do.

1. Find the wire paradox. Take an infinite straight wire, neutral in the laboratory frame S. Its positive lattice is at rest with line density $+\lambda_0$; its negative carriers have density $-\lambda_0$ and move along it at speed $u < c$. A positive probe charge q moves beside it at the same speed. Calculate its laboratory force. If an observer moving with the probe assumed the wire stayed neutral, why would the predicted force be zero?



Lattice at rest; carriers move right. Carriers at rest; lattice moves left.

The frame S' moves at speed u relative to S. Charge spacings are schematic, not to scale.

2. Ask what has to change. Maxwell's vacuum equations predict the speed of light. What happens to that speed under Galilean velocity addition? Investigate why this conflicts with requiring the same electromagnetic laws in all inertial frames. Study Einstein's two postulates and how Lorentz transformations replace Galilean transformations. Which assumption about time changes?

3. Build the space-time tools and resolve the paradox. Use Lorentz transformations to study simultaneity, length contraction, time dilation and velocity addition. Check what happens to a light pulse and to the space-time interval. Recalculate the two charge densities in S', comparing charge spacings at equal times in that frame. Is neutrality observer-independent? Find the electric force and compare both descriptions using the correct transformation of force.

4. See electricity and magnetism as one theory. Study how charge density and current transform together, and how electric and magnetic fields mix under a Lorentz boost. Apply these laws to the wire: compare fields obtained from the transformed sources with fields obtained directly from S. Why can the magnetic force vanish while the magnetic field remains nonzero? Introduce the four-current as a first relativistic object.

5. Test the picture on a moving point charge. Start with its Coulomb field at rest and transform to an observer who sees it moving uniformly. Draw or plot the electric and magnetic fields at one chosen time. How do the field strengths parallel and perpendicular to the motion change with speed? What becomes of the spherical symmetry?

Further directions.

- **Crossed fields and light.** For uniform perpendicular electric and magnetic fields, when can a change of observer remove one of them? Compare with a plane electromagnetic wave.
- **Covariant Maxwell equations.** Explore how four-vectors and the field tensor express the same laws in every inertial frame.

Expected outcome.

Prepare an illustrated account that follows the puzzle from the wire to relativistic space-time and back to electromagnetism. Include the main derivations, a quantitative comparison of the two wire descriptions and your field plots. Explain which classical assumptions fail and what the relativistic picture preserves.

Starting materials.

The starting puzzle. Feynman, Leighton and Sands, *The Feynman Lectures on Physics*, vol. II, section 13-6 for the wire. Chapter 26 continues with moving charges and field transformations. Check the unit conventions. Wire example.

Feynman, chapter 26.

Space-time foundations. OpenStax, *University Physics*, vol. 3, sections 5.1-5.6: the postulates, simultaneity, time dilation, length contraction and Lorentz transformations. Open textbook.

Putting the theory together. David Tong, *Electromagnetism*, chapter *Electromagnetism and Relativity*: four-vectors, sources, field transformations and moving charges. Use the field tensor sections for the optional continuation. Chapter PDF.

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